# Einstein's Mass-Energy Equation, Volume I: Early History and Philosophical Foundations

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# BOOK REVIEWS

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**Einstein's Mass-Energy Equation, Volume I: Early History and Philosophical Foundations.** Francisco Fernflores. 116 pp. Momentum Press, New York, 2018. \$49.95 (paper). ISBN 978-1-60650-857-2. **Einstein's Mass-Energy Equation, Volume II: Quantum Mechanics and Gravitation, Empirical Tests, and Philosophical Debates.** Francisco Fernflores. 93 pp. Momentum Press, New York, 2018. Price: \$49.95 (paper). ISBN 978-1-94664-674-3. (Eugene Hecht, Reviewer.)

*Einstein's Mass-Energy Equation* comprises two thin paperback volumes. The author, Dr. Fernflores, is a professor of philosophy at Cal Poly in San Luis Obispo, who brings a fresh and insightful perspective to his analyses. The work is a mathematically sophisticated treatise that will be most appreciated by philosophers of science, physicists, and educators, who have more than a passing interest in Einstein's splendid equation. *Einstein's Mass-Energy Equation* is a scholarly treatment, quite distinct from the run-of-the-mill trade books that have $E = mc^2$ emblazoned on their covers, and too often almost nothing redemptive inside. Interestingly, Fernflores discusses a number of philosophical issues associated with the concepts of mass and energy: are the two equivalent, interconvertible, conserved, and so forth. Alas, he is not a storyteller, and so the reader will not get much help in learning how the modern interpretation of $E = mc^2$ evolved; intense scrutiny of detail has led to a bit of myopia regarding trees, and not seeing the forest. Still, as a physicist, I especially enjoyed Fernflores's philosophical asides and insights.

As one might expect, the Preface spells out the author's intentions to present a "historical and philosophical analysis of the development and interpretation of Einstein's famous equation $E = mc^2$." That very first line should leap out at the informed 21st-century reader. Knowing that it was terribly ambiguous, Einstein only occasionally wrote his mass-energy equation as $E = mc^2$. When he did so, $E = mc^2$ had already become so iconic, it was almost unavoidable. Since $E$ has long been taken to be *total energy*—which includes *kinetic energy* (*KE*)—and since $E = mc^2$ *only applies when the system is at rest*, Einstein was generally careful to explicitly distinguish between $E$ and $E_0$, the latter being the *rest-energy* of the system. Accordingly, he most often wrote the great equation as $E_0 = mc^2$.

For the sake of those new to the intricacies of relativity theory, where it will contribute to clarity, I will remind the reader that retaining $E = mc^2$, wherein $E$ is not just rest-energy, requires that $m$ be something called "relativistic mass." Here and there in this review, that concocted quantity will then be symbolized as $m_R$ just to avoid confusion, albeit at the cost of a bit of clutter. Thus, an unambiguous rendering of $E = mc^2$ would be $E = m_R c^2$, though I don't expect that a billion T-shirts will be repainted any time soon.

Later in the Preface, Fernflores writes:

> *Similarly, there has been some controversy among physicists about how best to express Einstein's result in symbolic notation. Without wading into the details of this debate, let us simply note that there are two candidates for how to denote the energy in the mass-energy equation: $E$ and $E_0$. Similarly, there are two candidates for how to denote the mass: $m$ and $m_0$. There are, then, four possible ways to write down the mass-energy equation.*

I wish Fernflores had waded in with both feet, since what he takes to be a mere debate about nomenclature hides a far more profound disagreement about the very meaning of the entire proposition. Alas, without robustly dealing with the physics behind the nomenclature, Fernflores ties his treatment into conceptual knots, and leaves his readers adrift somewhere in the 1980s. Although he handles each separate piece quite well, the puzzle never comes together; we never get the satisfaction of arriving at the modern picture. What should have ended with the triumph of space-time, the fundamental equation $E^2 = m^2c^4 + \mathbf{p}^2c^2$, Lorentz invariant mass, $E = \gamma mc^2$, massless photons, and the Standard Model, ends instead with an unresolved whimper, and a box full of jumbled pretty pieces.

Einstein's famous equation, which we—following his lead—write as $E_0 = mc^2$, is not something one can derive directly from what might be called first principles. Instead, during his lifetime, Einstein produced around 18 different derivations—most of which are predicated on somewhat idealized, thought experiments. These analyses affirmed his seminal equation, though no such theoretical effort could prove it to be irrefutably true. Fernflores has provided his readers with a careful exegesis of four of those derivations in Volume I (1905, 1907, 1912, and 1935), and the last of them (1946) in Volume II. These are all gems that highlight Einstein's imaginative brilliance and unsurpassed analytic skills, and Fernflores does a worthy job of explaining them.

It's telling that Fernflores naturally enough begins his treatment (Section 1.1) with Einstein's second (September 1905) paper on relativity rather than his first. Yet it was in that initial article (June 1905) "On the Electrodynamics of Moving Bodies" that Einstein inadvertently contributed to the ultimate muddle surrounding $E_0 = mc^2$. In the late 1800s, several European physicists, principally H. Lorentz and M. Abraham, were working to show that mass was actually electromagnetic. After a titanic effort, Lorentz, whom Einstein greatly admired, came up with two speed-dependent equations for what came to be known as *transverse* and *longitudinal mass*. Young Einstein, almost as an afterthought, entered the fray, adding a last section to his June paper. Using his new relativity theory, Maxwell's equations, and $F = ma$ (a dubious place to start if you wish to show that mass is variable) Einstein, almost effortlessly produced two speed-dependent mass equations, one of which even matched one of Lorentz's results.

The concept that mass was entirely electromagnetic would soon enough be forgotten, but Einstein's little misstep (which he never bothered to rectify in print) would spawn a century of confusion and at times heated debate. Years later, people of the stature of Rutherford would misguidedly credit Einstein with discovering the speed dependence of mass (Vol. II, p. 28). And decades later, scholars of the caliber of Born, Pauli, Bohm, Bondi, Hawking, Penrose, and even Feynman, were still writing about the dependence of mass on velocity.

Much of Chapter 1, Volume I is an interesting discussion of "Mass: From Newton to Einstein." Fernflores begins with Newton who used the word *mass* (*massa* in Latin) as the equivalent of *quantity of matter* (quantitas materiæ in Latin). Sir Isaac actually favored the term "quantity of matter"—which goes back to Giles of Rome (c. 1243–1316)—and rarely used "mass," but that's not important here. What is important is that Fernflores repeats (Vol. I, p. 5) the widely held notion that Newton introduced the concept of inertial mass—he did not. That honor goes to Johannes Kepler (1571–1630) who gave us the conceit of "inertia" (albeit incompletely developed); the idea that inertia depended on mass (*moles* in Latin); and the insight that gravity was a mutual attraction that depended on the masses of the interacting bodies. Kepler's concept of mass (*moles*) was well known in the 1680s, and I suspect Newton used the alternative *massa* just to establish a distinction—in any event, he never credited Kepler.

After an informative discussion of mass as considered, in turn, by Newton, Euler, Maxwell, and Mach, Chapter 1 ends with a brief historically based discussion of energy and its conservation. Because Fernflores focused on history, he could overlook the tautological nature of the usual definition of energy in terms of the ability to do work—which is itself defined as a transfer of energy. It would seem that *energy is the ability to transfer energy*. So, we are left to press on without any useful definition of energy that accords with our modern-day physics. Since we will be talking about energy I suggest, as a starter, that *energy is the capacity of a physical system to effectuate change*.

Einstein's September 1905 paper "Does the Inertia of a Body Depend upon Its Energy Content?" gets a fairly detailed analysis in Section 2.2. The reader should notice that Einstein arranged to keep the body being analyzed in his thought experiment at rest, and so designated its energy as $E_0$; that zero represents zero speed. What Einstein derived is that "If a body releases the energy $L$ in the form of radiation, its mass decreases by $L/c^2$." Fernflores rightly reports that nowhere in this 1905 paper does any variation of $E = mc^2$ appear—that would have to wait until 1907. I suggest that the likely reason for this cautious approach was that Einstein had no way of knowing that there was no such thing as "dormant" mass, which might remain behind after all the energy of a body was removed. By 1907 he was willing to take the leap, and wrote, "It seems far more natural to consider any inertial mass as a reserve of energy." Much later (1935) he simply asserted, "one can stipulate that $E_0$ should vanish together with $m$."

Fernflores provides a detailed treatment of Einstein's 1906 and 1912 derivations. Keep in mind that in discussing the 1912 thought experiment, Fernflores tells us that Einstein required "that the mass of the object $m$ and its charge $\varepsilon$ are invariant"—that means $m$ is definitely not $m_R$. Though Fernflores does not mention it, Einstein, after assimilating the important 1906 paper by Planck, abandoned any notion that mass was speed dependent, for him $m$ forever after would be invariant; hence the symbol $m_0$ became extraneous; for Einstein *rest-mass is mass; there is only one mass, m*—no debate. Here (Vol. I, p. 60) Fernflores perceptively raises an intuitively troubling point that commentators in the Einstein invariant-mass school usually avoid. How is it that *internal KE* contributes to the mass of a compound system? Philosophical asides like this make the two books engaging even though Fernflores doesn't attempt to resolve this particular issue.

In Section 3.4 Fernflores provides a lengthy exegesis of an influential historical paper by Lewis and Tolman [Philos. Mag. 510–523 (1909)], and in so doing opens the proverbial can of worms. A year earlier G. N. Lewis (the physical chemist who coined the word photon) had published a piece on relativistic dynamics, concluding that light possessed mass. He used the classical formulation of momentum, $p = mv$ rather than the correct relativistic form $p = \gamma mv$, as derived by Planck, in which $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$, $\beta = v/c$, and $\gamma \geq 1$. That approach would later lead Lewis and Tolman to the remarkable conclusion that $m = m_0(1 - \beta^2)^{-\frac{1}{2}}$; here $m$ is the *relativistic mass* (our $m_R$) and $m_0$ is resurrected as *rest-mass*. Einstein's 1905 youthful nod to Lorentz, his two speed-dependent masses, now coalesced into this new creature, relativistic mass. Fernflores does a good job focusing on and analyzing the Lewis/Tolman result, $E = mc^2$, which reduces to $E_0 = m_0c^2$ when the system is at rest. Because $E = mc^2$, wherein $c$ is constant, this mass (i.e., $m_R$) is effectively equivalent to total energy, which itself varies with speed, hence $m$ must vary with speed. Unfortunately, Fernflores says nothing about the profound physical implications of all of this.

Notice that whereas the Einstein invariant-mass school showed that total energy is $E = \gamma mc^2$, the relativistic-mass school claimed instead that $E = mc^2$, where $\gamma$ is "inside" the $m$. In other words, this $m$ is $m_R$ where $m = \gamma m_0$, and so $E = \gamma m_0c^2$. Because the two opposing formulations for total energy, $E = \gamma m_0c^2$ and $E = \gamma mc^2$, are structured around the same $\gamma(v)$ there is as yet no experiment that has been able to distinguish between them.

The differences in the approaches might appear to be little more than a matter of nomenclature and hence merely debatable, as Fernflores implies. Nothing could be further from the truth! The Einstein invariant-mass school maintains that unlike the kinematical concepts of spatial and temporal intervals, mass is a dynamical concept independent of relative speed. For them total energy is the determinant of inertia. On the contrary, the relativistic-mass school claims that mass is a function of speed, approaching infinity as relative speed approaches $c$. For them mass is the determinant of inertia. The invariant-mass school takes individual photons to be completely massless ($m = 0$), whereas the relativistic-mass school generally takes individual photons to have mass ($m_R \neq 0$). Incidentally, to date, *no experiment has ever been able to detect even the slightest trace of mass for the photon.*

One usually arrives at relativistic mass by way of the momentum 3-vector, $\mathbf{p} = m\mathbf{v}$, which is valid only when $v \ll c$. That approach makes no use of the proper 4-velocity; it clashes with the modern space-time formulation of special relativity wherein $m^2 = (E/c^2)^2 - (\mathbf{p}/c)^2$. Why Fernflores chose to avoid the intricacies of this entire remarkable episode is not apparent, but he cannot fulfill his stated goal of presenting a "historical and philosophical analysis of the development and interpretation of Einstein's famous equation $E = mc^2$" without forthrightly dealing with them; the physics of $E = \gamma mc^2$ is not the physics of $E = mc^2$.

Volume I ends with a nice analysis of an important paper by Feigenbaum and Mermin [AJP, **56**, 18–21 (1988)] in which they derive the mass/energy equation from dynamical considerations. Volume II continues the effort focusing on the relationship between what Einstein called energy content [*Energieinhalt*], and gravitational mass. It begins with a well done and welcomed exegesis of de Broglie's 1924 paper "A Tentative Theory of Light Quanta." That's followed by a discussion of the famous Cockcroft/Walton experiment of 1932. In that study, the total mass of an incoming proton plus the mass of a target lithium nucleus, minus the masses of the two emerging alpha particles, resulted in a net decrease in mass. As predicted, that decrease turned out to be equivalent to the *KE* of the alpha particles. Note that all those masses are what were called rest-masses. Like many scholars, Fernflores claims the experiment confirmed $E = mc^2$ —well, not quite; it just as well confirmed $E_0 = mc^2$. Actually, the decrease in mass ($\Delta m$) made available an amount of energy ($\Delta mc^2$) that appeared as *KE*, independent of whether $E = mc^2$ (this $m$ is $m_R$) or $E_0 = mc^2$ (this $m$ was once $m_0$). The chapter ends with a discussion of the beautiful neutron capture experiment of S. Rainville *et al.* [Nature **438**, 1096–1097 (2005)] which led to a far more precise confirmation.

Section 3.1 tackles the philosophical question of whether mass and energy are "equivalent" or "interconvertible." It begins with a discussion of a paper by Bondi and Spurgin [Phys. Bull. **38**, 62–63 (1987)] who were then among an only gradually diminishing number of physicists arguing in favor of relativistic mass. They maintained that all energy has mass, and that energy and mass are independently conserved. Photons have energy and so photons must have mass. Strangely enough, Fernflores "underwrites Bondi and Spurgin's claim that, in a sense, mass and energy are different quantities" by pointing out that the photon is an object that has energy but not mass.

In the penultimate chapter titled "Contemporary Debates and Insights," Fernflores chooses to discuss a paper by R. Baierlein [Phys. Teach. **29**, 170–175 (1991)], a scholar seemingly in the relativistic-mass school. Baierlein argued that mass and energy are not "convertible," whereas matter and radiation are. And he tells us that "*Matter* is tangible stuff, what you can hold in your hand." A neutrino is matter, though I'm not so certain anyone will ever hold one in hand. That rather simplistic definition leads to what the philosopher Fernflores calls "category mistakes." Be that as it may, I would simply point out that many distinguished physicists including Schrödinger, Born, de Broglie, Pauli, Schwinger, Wilczek, Synge, Okun, and countless others, have taken the position that light is the most ephemeral form of matter. Indeed, quantum field theory treats all quantum particles equally, whether they manifest mass or not. It is then hardly a great revelation to conclude that one form of matter can be "transformed" into another form of matter; just as one form of energy can be "transformed" into another form of energy. As Einstein put it, "inert mass is simply latent energy." And "[m]ass and energy are therefore essentially alike; they are only different expressions of the same thing." Perhaps the deeper question is what, if anything, happens during those transformations?

Touted as being more appealing pedagogically, the relativistic-mass approach came to prevail in our textbooks and universities by the mid 20th century. That situation began to very slowly change, in part driven by the post-war renaissance in particle physics. In 1963 Taylor and Wheeler published their excellent book *Spacetime Physics*, which is a thoroughly modern treatment that helped to bring invariant mass back to the fore. A highly influential 1989 article in *Physics Today* (June, 31–36), by the Russian theoretician Lev Okun (he coined the term *hadron*) focused attention on the $E = mc^2$ debate and marked something of a turning point. It was Okun who first pointed out that there are (in Fernflores's words) "four possible ways to write down the mass-energy equation," but Okun, unlike Fernflores, argued that only one of them was both unambiguous and correct. By the early 21st century the splendid successes of the Standard Model provided powerful physical reasons to embrace both the *massless photon* and *invariant mass*—the great debate seems to be essentially over, even if there are still vociferous holdouts.

Even so, it should be appreciated that both schools have their little conceptual challenges. For example, if you embrace relativistic mass you have to live with the fact that a high-speed particle, say, an electron, impacting another electron which is at rest, can transfer all of its momentum to that particle. If mass is a function of speed, such a collision ought to resemble a bus colliding with a bug. On the other hand, if you embrace invariant mass, you have to accept that a collection of individually massless photons, distributed such that it has zero net momentum, nonetheless has rest-energy and hence mass. There is no zero-momentum frame for a collimated beam of photons and so it has no mass. By contrast, a cloud of identical photons traveling in every direction, such that its net momentum is zero, has mass.

I cannot know why Fernflores chose to ignore all of this; besides being crucial to his outlined program, the saga does illuminate the way physics advances—hardly ever without dispute. Nonetheless, I do fault him for not including in his bibliography any of the plethora of important published papers relating to the great mass/energy debate. We can easily overlook the books' other tiny editorial flaws—lost punctuation marks, words hyphenated that shouldn't be, others missing or misplaced here and there (Vol. I, pages 1, 25, 31, 51, 69, and 72, and Vol. II, page 19), and $E_2{}^2/c^2$ when it should have been $E_2/c^2$ on page 20 of Vol. II. By the way, it's William John Macquorn Rankine (Vol. I, p. 18).

Fernflores's treatments of Einstein's selected papers are well crafted, edifying, and interesting. The two books that comprise *Einstein's Mass-Energy Equation* are much enhanced by the addition and analysis of the several classic papers by de Broglie, Lewis and Tolman, and others. Philosopher Fernflores takes up physical and metaphysical issues that are rarely discussed in print. In a scholarly way Fernflores often boldly goes where no one has gone before, and that's appreciated. The two small books could enrich any college-level course on Special Relativity. All in all, shortcomings aside, *Einstein's Mass-Energy Equation* is a fine contribution to the literature, one that provides both a convenient source of foundational material and a stimulating read.

*Eugene Hecht is the author of a number of books, including three on the American ceramic artist George E. Ohr, and seven on physics. Among the latter is Optics, published by Addison-Wesley. Professor Hecht's most recent physics book is Quantum Mechanics, published by Schaum's Outline, McGraw-Hill. His main interests are the history of ideas and the elucidation of the basic concepts of physics. He spends most of his time teaching, studying physics, and training for his sixth-degree black belt in Tae Kwan Do.*

## BOOKS RECEIVED

**The Tectonic Plates are Moving!** Roy Livermore. 482 pp. Oxford U.P., New York, 2018. Price: $32.95 (hardcover) ISBN 978-0-19-871786-7.

**A Student's Guide to Atomic Physics**. Mark Fox. 291 pp. Cambridge U.P., New York, 2018. Price: $24.99 (paper) ISBN 978-1-108-44631-0.

**Energy, Entropy, and the Flow of Nature**. Thomas Fairchild Sherman. 282 pp. Oxford U.P., New York, 2018. Price: $45 (hardcover) ISBN 978019069534.

**Multilayer Networks: Structure and Function**. Ginestra Bianconi. 408 pp. Oxford U.P., New York, 2018. Price: $75 (hardcover) ISBN 978-0-19-875391-9.

**Understanding Stellar Evolution**. Henry J. G. L. M. Lamers and Emily M. Levesque. 305 pp. IOP Publishing, Bristol, UK, 2017. Price: $150 (hardcover) ISBN 978-0-7503-1279-0.

**The Elements of Relativity**. David M. Wittman. 322 pp. Oxford U.P., New York, 2018. Price: $32.95 (paper) ISBN 978-0-19-965864-0.

**Aperiodic Crystals: From Modulated Phases to Quasicrystals: Structures and Properties (2nd ed.)** Ted Janssen, Gervais Chapuis, and Marc de Bossieu. 550 pp. Oxford U.P., New York, 2018. Price: $60 (paper) ISBN 978-0-19-882442-2.

**Subnational Hydropolitics: Conflict, Cooperation, and Institution-Building in Shared River Basins**. Scott M. Morse. 282 pp. Oxford U.P., New York, 2018. Price: $85 (hardcover) ISBN 9780190864101.

**Modern Thermodynamics for Chemists and Biologists**. Dennis Sherwood and Paul Dalby. 899 pp. Oxford U.P., New York, 2018. Price: $60 (paper) ISBN 978-0-19-878470-8.

**Lagrangian & Hamiltonian Dynamics**. Peter Mann. 552 pp. Oxford U.P., New York, 2018. Price: $38.95 (paper) ISBN 978-0-19-882238-7.

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